Name: _____

MA 1118 - Multivariable Calculus
Quiz 5 - Quarter I - AY 02-03

Instructions: Work all problems. Read the problems carefully. Show appropriate work, as partial credit will be given. No notes or tables permitted.

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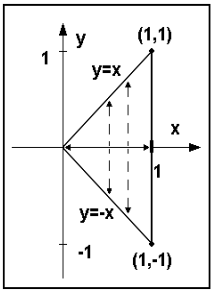
1. (20 points) Integrate the function

$$f(x,y) = x + y$$

over the triangular region with vertices at $(0,0)$, $(1,1)$ and $(1,-1)$.

solution:

The region described is shown below



From this figure, it should be obvious that the best way to cover this region is to "slice" it vertically, i.e. for each x between 0 and 1, let y vary between $y = -x$ and $y = x$. This leads to the integral

$$\int_{x=0}^{1} \underbrace{\int_{y=-x}^{x} (x+y)\,dy}_{=(xy+\frac{y^2}{2})\,\Big|_{y=-x}^{x}}\,dx$$

But $\quad (xy + \dfrac{y^2}{2})\,\Big|_{y=-x}^{x} = \left(x(x) + \dfrac{(x)^2}{2}\right) - \left(x(-x) + \dfrac{(-x)^2}{2}\right) = 2x^2$

Therefore

$$\int_{x=0}^{1}\int_{y=-x}^{x} (x+y)\,dy\,dx = \int_{x=0}^{1} \left(2x^2\right) dx = \frac{2x^3}{3}\bigg|_0^1 = \frac{2}{3}$$